

Code: 20BS1101

**I B.Tech - I Semester – Regular / Supplementary Examinations
FEBRUARY - 2023**

**CALCULUS AND LINEAR ALGEBRA
(Common for ALL BRANCHES)**

Duration: 3 hours

Max. Marks: 70

Note: 1. This paper contains questions from 5 units of Syllabus. Each unit carries 14 marks and have an internal choice of Questions.
2. All parts of Question must be answered in one place.

BL – Blooms Level

CO – Course Outcome

			BL	CO	Max. Marks
UNIT-I					
1	a)	By reducing it to Normal form find the rank of the following matrix. $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$	L3	CO2	7 M
	b)	Investigate for what values of a, b the equations $x + y + z = 6, x + 2y + 3z = 10,$ $x + 2y + az = b$ have (i) no solution (ii) a unique solution (iii) many solutions.	L4	CO4	7 M
OR					

2	a)	By reducing it to Echelon form find the rank of the following matrix. $\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$	L3	CO2	7 M
	b)	Solve the system of equations $x + y + 2z = 4, 2x - y + 3z = 9,$ $3x - y - z = 2$	L3	CO4	7 M

UNIT-II

3		Verify Cayley –Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$ Calculate A^4 and A^{-1} using Cayley-Hamilton theorem.	L3	CO2	14 M
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OR

4	a)	How would you find eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{bmatrix}$	L4	CO4	7 M
	b)	Calculate $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$	L3	CO2	7 M

UNIT-III

5	a)	How would you confirm Rolle's theorem for $f(x) = e^x(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$	L3	CO5	7 M
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	b)	Discover 'c' of Cauchy's mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ in $[a, b]$, $0 < a < b$	L3	CO5	7 M
OR					
6	a)	Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$	L3	CO6	10 M
	b)	Obtain the Maclaurin's series expansion of $\sin x$.	L3	CO6	4 M
UNIT-IV					
7	a)	If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ then calculate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	L3	CO3	7 M
	b)	Discover the extreme values of the function $x^4 + y^4 - 2x^2 + 4xy - 2y^2$	L3	CO5	7 M
OR					
8		Calculate the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	L3	CO3	14 M
UNIT-V					
9	a)	Appraise $\int_0^{\frac{\pi}{4}} \int_0^{\sin\theta} \frac{r \, dr \, d\theta}{\sqrt{a^2 - r^2}}$	L4	CO5	7 M
	b)	Change the order of integration and evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy \, dx$	L4	CO3	7 M
OR					

10	a)	Solve $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx dz$.	L3	CO5	7 M
	b)	By concluding the limits of integration find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$	L4	CO3	7 M